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The meaning of absurdity[1]

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Introduction

There are two adversarial views on the foundation of meaning. Referentialism claims that the basis of meaning is referential semantics while inferentialism holds that meaning is based on inferential rules. The latter has loose affinities with the Wittgensteinian slogan 'meaning is use'.

Timothy Williamson captures the dispute between referentialism and inferentialism by saying that the difference is the direction of explanation: 'referential[ism] gives center stage to the referential semantics for a language, which is then used to explain the inference rules for the language, [...] as those which preserve truth [...]'. Inferentialism, on the other hand, starts off with inferential rules 'which are then used to explain its referential semantics, [...] as semantics on which the rules preserve truth'. He adds that these directions cannot be combined because it would cause an obvious circularity in the explanation. (Williamson 2009, 137.)

It could be said that the common ground for both directions is truth conditions for connectives. Referentialism builds compositional semantics which yield truth conditions for connectives and inferentialism gives us inferential rules which confirm truth conditions. In this context, Panu Raatikainen claims that

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referentialism has the upper hand. He utilises Carnap's considerations to reach this conclusion. Raatikainen gives Carnap's considerations a rather unique flair as he aims to convince the superiority of referentialism with Carnap's results. In short, Raatikainen sees Carnap's considerations especially problematic for inferentialism. He says that model-theoretically you can come up with valuations that fail to meet the truth conditions to which *both* accounts subscribe. The crux of the discussion involves ruling out these valuations. Raatikainen claims that referentialism can rule out these evaluations while inferentialism cannot.

Carnap's problem as a problem for inferentialism

The truth condition for negation

$$\text{(NEG)} \quad \neg\, \text{A is true} \quad \Longleftrightarrow \quad \text{A is false}$$

is an essential principle in both classical and intuitionistic logic (Raatikainen 2008, 282–284 and Murzi and Hjortland 2009, 481). Carnap has shown a nonnormal model which violates this principle: For any sentence A, both A and ¬A are true. Raatikainen argues that Carnap's problem poses 'a real challenge' for inferentialists like Dummett and Prawitz. On the one hand, they hold that the rules of inference determine the meanings of logical connectives but, on the other hand, they adhere to NEG. 'Yet the standard formalisations of logic (rules of inference) do not rule out non-normal interpretations which violate these principles' (Raatikainen 2008, 284). To illustrate, let there be a classical propositional logic (CPL) in which a set of valuations V for sentences and connectives is produced in a standard recursive manner. Consider an expansion of CPL in the following way. Let there be a set of admissible valuations $V\cup\{v^*\}$ (where for any A, $v^*(A) = T$). As Julien Murzi and Ole Thomassen Hjortland explain, both semantics yield the same (semantic) consequence relation: $\Gamma \vDash_V A$ iff $\Gamma \vDash_{V\cup\{v^*\}} A$. (Since $\vDash_{V\cup\{v^*\}}$ provides no counterexample for $\Gamma \vDash_V A$. Furthermore, assuming $\Gamma \nvDash_V A$, then there is a valuation $v \in V$ according to which every member of Γ is true and yet A is false. Because $v \in V\cup\{v^*\}$, the very same valuation is also in the extended set of valuations. Hence, $\Gamma \nvDash_{V\cup\{v^*\}} A$.)

As a consequence, the formalisation of classical propositional logic, $\vdash_{CPL}$ is sound and complete with respect to standard semantics $\vDash V$ iff it is sound and complete with respect to $\vDash_{V\cup\{v^*\}}$. The problem is that the inferentialist cannot make a distinction between the semantics on the basis of soundness and completeness results. Yet there is a big difference. In $\vDash V_{\cup\{v^*\}}$, NEG 'fails massively'. (More elaborate exposition in Murzi and Hjortland 2009, 480–481.) Raatikainen concludes that inferentialism owes us an explanation as to how the problem is circumvented (Raatikainen 2008, 287).

It should be pointed out that the troublesome valuation surely affects other connectives too. For example, it is a rather intuitive thought that 'A ∧¬A' is false but with the non-standard valuation this comes out as true. It is also an intuitive thought that 'A → ¬A' is false when A is true but that is not the case with $\vDash_{V\cup\{v^*\}}$. However, NEG is the only truth condition that directly relies on the fact that A and ¬A cannot be true at same time. That is why NEG deserves special attention.[2]

Murzi and Hjortland's intuitionistic solution

Murzi and Hjortland stress that Raatikainen's dismissal of Dummett and Prawitz is too quick and they establish an inferentialist response based on Dummett and Prawitz's work. I claim that Murzi and Hjortland do not provide a solution to the problem, at least not without serious caveats. I then introduce my solution which is based on Neil Tennant's work. The solution takes Tennant's adherence to paraconsistency seriously. On the basis of this, I offer a solution which explicates negation with the principle of consistency. The key ingredient in the solution is that absurdity is viewed as a primitive expression of the principle of consistency. Finally, Murzi and Hjortland are sceptical about a bilateralist solution. I go on to show that the developed view can contribute to the bilateral solution. I argue that if a bilateralist adopts a similar paraconsistent view, then Carnap's problem is not a threat to her, contrary to Murzi and Hjortland's claim. Initially, one might object that the problem arises because of the non-normal valuation and it should be ruled as inadmissible in the first place since it violates NEG. But as Murzi and Hjortland point out, this misses Raatikainen's point:

> [I]f meanings are to be determined by the inferential rules, and if meanings are truth-conditions, logical inferentialists cannot legitimately appeal to NEG [...], on the pain of invoking a previous knowledge of the meanings they are trying to capture (Murzi and Hjortland 2009, 481).

In short, an inferentialist cannot appeal to semantics to justify the inferential rules. It should be the other way round. The situation that both A and ¬A are true has to be ruled out with the inferential rules which ultimately determine the truth-conditions such as NEG. Only after this, the inferentialist can commit to NEG.

First, intuitionists equate truth with proof. Thus, the investigation is narrowed to excluding the possibility that there is a case where both A and ¬A are provable (Murzi and Hjortland 2009, 483). Essentially, Murzi and Hjortland's solution to the proof-theoretic version of Carnap's problem relies on two points:

On canonical proof: A proof whose last step is an introduction rule.

On Prawitzian view on absurdity: The introduction rule for ⊥ is null.

They introduce Brouwer-Heyting-Kolmogorov (BHK) clauses to specify proofconditions for complex statements. BHK clauses define ¬A as A →⊥. (Murzi and Hjortland 2009, 483.) Given BHK clauses, Carnap's situation is that A is proven and

[2] Raatikainen also discusses disjunction at length but I think Murzi and Hjortland's response to that is satisfactory so here I am concentrating solely on negation. (Raatikainen 2008, 283–284 and Murzi and Hjortland 2009, 483–484.)

that ¬A is also proven. However, the latter is blocked with Prawitz's view of absurdity. According to him, the meaning of ⊥ is determined by a null introduction rule and the elimination rule is absurdity rule:

$$(\bot\text{ -E})\,\frac{\bot}{A}$$

in which A can be substituted with any atomic sentence of the language (Prawitz 1973, 243). Murzi and Hjortland argue that the null introduction rule makes sure that any proof of A → ⊥ cannot satisfy the notion of canonical proof. Because the introduction rule is null, there simply is not a way to use the rule as a last step in a proof. (Murzi and Hjortland 2009, 483–484.) There seems to be something rather odd with this response. It seems as if Murzi and Hjortland are saying that there is no way to introduce negation because the rule for the introduction of absurdity is empty. But concerning the introduction of ¬A, the introduction rule for absurdity is the wrong rule. The (intuitionistic) rules for negation are

$$(\neg\text{-I})\quad \frac{\begin{array}{c}A\\ \vdots\\ \bot\end{array}}{\neg A} \qquad\qquad (\neg\text{-E})\quad \frac{A \quad \neg A}{\bot}$$

An inferentialist can introduce negation (and thereby prove ¬A in canonical way) with ¬-I rule. For surely there must be a way to prove negated claims within the inferentialist system.[3] The upshot is that there is a perfectly good way to introduce ¬A. The real question is whether you can introduce ¬A (with ¬-I) given that A is already proven.

Murzi and Hjortland are right in their contention the conception of absurdity is important in solving Carnap's problem but they should have concentrated on explicating the connection between negation and absurdity.

Hand's criticism of Dummett

Harmony between the introduction and elimination rules guarantees that nothing is added to (or left out from) the elimination rule (in respect to the relevant introduction rule). The elimination rule can only unpack the information that the introduction rule packed in. (e.g. Rumfitt 2000, 782–792). Neil Tennant criticises Prawitz's conception of absurdity because the question about harmony cannot be properly investigated. He says that it is unnatural that an introduced concept has only an elimination rule,

3 I owe this point to the referee(s). The referee(s) also point(s) out that Murzi and Hjortland's solution applies only to proofs, not to deductions in general. The referee(s) go(es) on to point out that, surely, NEG applies to all sentences, not just proofs. In my view, this points to the fact that Murzi and Hjortland's solution is not entirely satisfactory. However, I am willing to disregard this asymmetry and concentrate on the more serious problem that Murzi and Hjortland place too much weight on the wrong rule.

'with no introduction rule to which it is genuinely answerable' (Tennant 1999, 216). Dummett also sees this problematic. He says that it is a usual practice not to impose an introduction rule for ⊥. He suspects the motivation for this is an implicit appeal to principle of consistency. In order to harmonise the introduction and elimination rules for ⊥, Dummett proposes the following introduction rule:

$$(\bot\text{-I}) \quad \frac{A \quad B \quad C \quad \cdots}{\bot}$$

where A, B, C, ...are atomic sentences of the language. The idea is that the premise set includes all the atomic sentences of the language. Dummett comments: 'The constant sentence ⊥ is no more problematic than the universal quantifier: it is simply the conjunction of all atomic sentences'. This harmonises the elimination rule from which one can infer any atomic sentence of the language.[4] At the same time, the intuitive thought is that no language is consistent and you are bound hit inconsistency at some point. However, Dummett himself observes that the intuitive thought is beside the point. As far as logic is concerned, a language *L* might be consistent. Dummett thinks that the principle of consistency is not a logical law. (Dummett 1993, 295–296.)

In his comments on Dummett's proposal, Michael Hand makes much of Dummett's point that a language need not to be inconsistent. He first admits that Dummett's introduction and elimination rules do explicate the meaning of ⊥ in a harmonious way:

> The answer to the question of the meaning of ⊥ is now obvious: it has precisely the same logical power that a conjunction of all atoms other than ⊥ would have, if we had infinitary conjunction in the language. (Hand 1999, 189).

But then he goes on to criticise the proposal:

> One's first reaction to this observation should be to balk. Surely there is something wrong if we cannot fix the truth-condition, not to mention the meaning, of ⊥ any better than this. The constant ⊥ is supposed to be false, and if meaning is use, then our rules governing ⊥ had better make it so. What Dummett points out is that the intuitionistic rules cannot even prevent ⊥ from meaning something that might be true [...] To put it differently, [the question with ⊥ is] why are we unable to formulate rules ensuring that

[4] The adding of an introduction rule for ⊥ of course undermines Murzi and Hjortland's solution but that is not the issue because it already turned out that the rules for negation should have been the real issue in their solution. Also given that Dummett adds the introduction rule, it is odd that Murzi and Hjortland insist that they show that Carnap's results are not a problem for Dummett since their solution rests on the absence of the introduction rule.

> assignments meet the consistency condition, i.e. that a sentence and its negation are not both true?

In the present context, Hand's observation anticipates Carnap's problem. Since the essence of the problem is that according to the assignment both A and ¬A are true, Hand's point only emphasises Raatikainen's claim: The current inferentialist conception of ⊥ does not rule out Carnap's problem. In the following, I will introduce an alternative way to view absurdity. I claim that the view has a central role in the solution to Carnap's problem.

Absurdity based on semantics

The solution to Carnap's problem which I am offering is based on Tennant's thinking. But before going into Tennant's thinking in detail, I will make a clarifying point concerning Hand. He advocates a semantic view of absurdity. He starts his exposition with an observation that a false sentence does not mean that it is equivalent with a conjunction of all atoms. He continues: 'To say that a sentence is false is to say something much worse.' (Hand 1999, 192). The challenge is to frame this badness and the inferential rules alone cannot explicate the badness of falsity. Hand makes the following proposal:

> False sentences are bad because they fail. This failure is a semantical phenomenon, and purely intralinguistic rules cannot be formulated to characterise it. Intralinguistic rules can be formulated for contradictions, of course: if a person asserts one, reject it immediately. Nonetheless, this rejection is based on the realisation of a semantical fact about the claim, viz., that it is bound to fail. (Hand 1999, 194.)

Hand proposes that falsity is based on pragmatic and normative obligation. To avoid to assert something which turns out to be false is the primary obligation of an assertor. Importantly, this obligation can only be framed semantically, not inferentially. Hand presents a debate concerning his dachshund in the backyard:

> The important point is that this obligation cannot be explained except in overtly semantical terms, as far as I can see. When I said that my dachshund was in the back yard, you looked for him. You sought the referent of the name, to see whether it satisfied the predicate. Your discovery was that it did not, and the fact that it did not is just what makes it the case that I failed in my linguistic obligation to avoid falsehoods. (Hand 1999, 197.)

This *semantic* realisation gives us the principle of consistency, that A and ¬A are incompatible. The fact that this realisation is semantic seems to be another point for referentialism. At this point, the exact accusation against Murzi and Hjortland could be phrased that they aimed to provide a solution to Carnap's problem which then turned out to rest on insufficient explication of absurdity. According to Hand, the explication needs to be supplemented with a *semantic* explanation why a false sentence is a bad thing. However, I do not think this is insurmountable for inferentialism. I agree with Hand that something beyond inferential rules is needed but I do not think semantics is the only place to look for this.

In sum, Murzi and Hjortland's proposal is disappointing for two reasons. First, they do not pay enough attention to the rules for negation. Secondly, their view on absurdity is insufficient. In the following, I will bring clarity to both points. First an alternative conception of absurdity is introduced. This conception respects the principle of consistency and the inferentialist order of explanation. Then I will use this conception to clarify the notion of negation in the sense that the rules for negation provide a solution to Carnap's problem.

Tennant's paraconsistency, concept mastery and intuitionistic solution

Tennant is also concerned with the badness of absurdity: 'The source of the 'badness' that ⊥ seeks to register is contrariety' (Tennant 1999, 216). He thinks that contradiction 'is a matter of deep metaphysical necessity' (Tennant 2004, 362). According to Tennant, this separates his view from Dummett's:

> Whereas Dummett seeks a logical basis for metaphysics, I think we need, at this point, to put it the other way round. One needs a metaphysical basis for logic, insofar as we seek an origin for our grasp of the meaning of negation. I believe this is to be found in our sense of contrariety [...] (Tennant 1999, 217.)

Tennant sees the order of explanation as a crucial matter. It is my contention that Tennant does agree with Hand in this. Tennant sees ⊥-I and ⊥-E rules as a *logical* explication of absurdity but neither for Hand nor for Tennant that will do. Hand thinks that at the heart of falsity is a semantic explanation why false sentences are bad. Tennant thinks that the proper way to go is to explicate badness of absurdity metaphysically. At the same, both views make a distinction to Dummett. However, I suggest that there is a third way which looks for the basis of absurdity elsewhere but, in broad terms, stays faithful to Dummett. If we look at Tennant's 'metaphysics' more carefully, we can see that he does not go very far from Dummett. To elaborate, it seems to me that there are two versions of Tennant's view. The first view he presents in 'Negation, Absurdity and Contrariety'. He notes that the consistent language which Hand toys with is not actually learnable. According to Tennant, contraries among atomic sentences are crucial in learnability. Our grasp of different concepts depends on their patterns of instantiation, i.e. the grasp of concepts is based on different extensions. (Tennant 1999, 216-218.) In my view, this is not a very good *rebuttal* of Hand's overall point. Hand's point is that the inferentialist conception of absurdity needs to be supplemented with a semantic explanation and Tennant just seems to confirm this. I think his second proposal is better. The second view emphasises concept mastery. In 'An Anti-Realist Critique of Dialetheism', Tennant holds that some antonym-pairs derive from the structure of our phenomenology (Tennant 2004, 362.) For example, any competent speaker knows that an object cannot be solidly red and solidly green at the same time. For this reason, any competent language user can make the transition from Hot, Cold to ⊥. Tennant says that this realisation stems from the mastery of 'hot' and 'cold'. A child can learn what 'cold' means without knowing what 'not-hot' means. He says that contraries that he is talking about differ from sensory experience in that they are *a priori*. That is why the contraries do not have much to do with acquisition, i.e. sensory experience and 'everything to do with mastery'. (Tennnant 2004, 361–362.) Finally, you can ask where does this mastery stem from. In the present context, there are two possibilities: semantics and the inferential rules. According to Tennant's first story, the basis of absurdity is semantic. Realisation of absurdity is based on extension of contrary concepts like 'hot' and 'cold'. In the second story, this part is open. So the second story can accommodate inferentialism. The mastery of 'hot' and 'cold' could be explained with the inferential patterns concerning these concepts. In this case, no reference to semantics is needed. It seems to me that this kind of explanation is still compatible with Dummett's inferentialist conception of concept mastery (albeit it might not be compatible with Dummett's view of absurdity).

It is essential to understand the contrast between Hand's proposal and Tennant's proposal regarding the order of the explanation. Hand thinks that a false sentence is the primitive notion and this is then intimately connected with negation as NEG involves falsity. According to Hand, a contradiction is just a special case of a false sentence. It is always false. In distinction, Tennant thinks that contrariety is the primitive notion and then 'the conception of contrariety is expressed by means of an inferential transition from the contraries in question to absurdity' (Tennant 2004, 363). After this, Tennant moves on to explicate negation with the usual (intuitionistic) rules (re-introduced as a reminder):

$$(\neg\text{-I}) \quad \frac{\begin{array}{c} A \\ \vdots \\ \bot \end{array}}{\neg A} \qquad\qquad (\neg\text{-E}) \quad \frac{A \quad \neg A}{\bot}$$

It still remains to be seen how Tennant's view differs from Dummett's account and how Tennant's view provides a solution to Carnap's problem. Tennant is a relevantist and an essential part of his relevantism is paraconsistency, characterised as a rejection of absurdity rule, i.e. ⊥-E rule.[5] Tennant sees ⊥ as a (structural) punctuation mark. It represents a logical dead-end. (Tennant 1999, 200–205.) Since the sign does not have any propositional content, it is not subject to introduction or elimination rules.

To start the positive contribution of Tennant's paraconsistency, let us make the following observation. On the basis of Tennant's paraconsistent understanding of absurdity, we should not need any logical explanation of the badness. Whenever absurdity is derived, we should shout 'enough already' (Tennant 2004, 358). That is the point of ⊥ and there is no need to show any additional *logical* badness of ⊥ with the absurdity rule. The badness is in the derivation of absurdity itself. Tennant's version of paraconsistency appeals to the principle of consistency: it cannot be consistent to assert A and ¬A at the same time. When absurdity sign appears in ¬-I and ¬-E rules, it precludes any explicit definition of negation. It does not yield any propositional content to the definition of negation (such as A →⊥). Instead, the rules for negation give us instructions how to use negation in an inference.

Given all this, the solution to Carnap's problem was hidden in plain sight all along. ¬-E states that to claim that A and ¬A are both true leads to a *logical* dead-end. The contrast to Dummett's ⊥-I and ⊥-E rules is that Dummett's rules allow to equate ⊥ with the conjunction of all sentences of language but it does say anything about the semantics of the language. As far as the rules for ⊥ are concerned, all of the sentences might be true. Whereas, with Tennant's conception: 'There is no question – the possibility simply cannot arise – of ⊥ […] ever being true. And that is why negation works in such a way that it could never be the case that both P and ¬P were true.' (Tennant 2004, 364.) This gives the inferentialist the armoury to respond to the referentialist (or anyone) who proposes $V \cup \{v^*\}$ as an admissable valuation. The inferentialist can point out that *from the inferentialist point of view* the valuations are highly problematic since the valuations allow that A and ¬A are both true and this is an absolute logical dead-end. Most importantly, the inferential rules for negation can be viewed as meaning constituting rules so that they yield NEG as truth conditions for negation in a standard way.

Paraconsistency and bilateral solution to Carnap's problem

It is clear that Rumfitt's bilateralism aims to justify classical logic but he does this in an unusual way. As Raatikainen points out, usually referentialism comes with a realist notion of truth, i.e. evidence-transcendent notion of truth and semantic anti-realist like Dummett and Prawitz adhere to warranted assertability (Raatikainen

5 Here paraconsistency is understood as just that and nothing more. It has to made clear that Tennant's relevantism or paraconsistency is not 'inconsistency-friendly' in that it claims that not all inconsistencies lapse into absurdity. On the contrary, Tennant claims that all contrarieties do lapse into absurdity but he claims it without an appeal to absurdity rule because Tennant does not hold that the badness of absurdity is that it entails everything. More on this below.

2009, 282–283). It is precisely the adherence to the correspondence notion of truth which justifies classical logic. Because truth is not up to us, we can consider truth to be bivalent. This bivalence then justifies the crucial classical rules like Law of Excluded Middle (LEM) and Double Negation Elimination rule (DNE) even in undecidable discourse and even if DNE is non-harmonious by the inferentialist standards. Rumfitt's bilateralism provides a novel defence for classical logic. It concedes that truth might be equated with warranted assertability and hence it 'concedes the anti-realist standards for the justification of rules of inference', as Imogen Dickie points out (Dickie 2010, 163). This is the novelty in bilateralism: to admit the anti-realistic starting point in inferentialism and to justify classical logic anyway. I think Rumfitt maintains this strategy in his "Yes' and 'No" (Rumfitt 2000, 781-824). In a later response to Dummett's criticism, Rumfitt somewhat retracts this position. In his 'Unilateralism Disarmed: A Reply to Dummett and Gibbard' (2002, 305–321), Rumfitt writes:

> The oddity only arises, however, if truth is equated with the correctness of assertion and falsity with the correctness of denial; and I accept neither of these equations as generally correct theses about truth and falsity. For both Dummett and me, the notion of correctness is epistemic: to say that it is (objectively) correct to assert (or to deny) a sentence A is to say that knowledge is (tenselessly) available which, were a speaker to apprehend it, would warrant him in asserting (or in denying) A. As Dummett's reply makes clear, he wishes to equate, always and everywhere, the truth of a sentence with its being correct to assert it. I allow that there may be theories for which this conception of truth is correct; in the original paper ["Yes' and 'No"] I cited elementary arithmetic as a possible example. (Rumfitt 2002, 313.)

It seems to me that Rumfitt is proposing some form of truth pluralism here: Sometimes truth is an epistemically constrained notion and sometimes it is not. However, in my view this is a retrograding step as it takes the novelty out of bilateralism. The interest in bilateralism rests on the fact that it admits the anti-realistic starting point, truth can be equated with warranted assertability, and it still aims to justify classical logic. As soon as Rumfitt admits that truth is evidence-transcendent, bilateralism becomes redundant as the usual defence for classical logic is also available. That is why I will assume that Rumfitt adheres to warranted assertability or epistemically constrained notion of truth. At the very least, I am restricting the discourse under discussion to arithmetic, i.e. to discourses to which warranted assertability applies even by Rumfitt's standards.

That being said, Murzi and Hjortland are sceptical whether a classical bilateralist like Rumfitt is equipped to cope with Carnap's problem. Bilateralism recognises, in addition to assertion, an act of rejection. These acts are introduced to the formalisation as + A (assertion of A) and – A (rejection of A). Rumfitt forms the introduction and elimination rules for negation accordingly:

$$(+\text{-}\neg\text{-I})\ \frac{-A}{+(\neg A)} \qquad (+\text{-}\neg\text{-E})\ \frac{+(\neg A)}{-A}$$
$$(-\text{-}\neg\text{-I})\ \frac{+A}{-(\neg A)} \qquad (-\text{-}\neg\text{-E})\ \frac{-(\neg A)}{+A}$$

The main idea behind these rules is that they yield DNE in a way that satisfies the demand for harmony. Given +-¬-E and ¬¬A, we have +(¬¬A) ⊢ –(¬A) and given – -¬-E, we have –(¬A) ⊢ +A. Thereby, we have DNE: the rules take you from the assertion of ¬¬A to assertion of A.

Murzi and Hjortland suggest that the bilateralist's attempt to appeal to these rules fail to block Carnap's problem. Let +A and –A be signed formulae for any A ∈ WFF (well-formed formula) and let WFF_{sign} be the set of signed formulae. In this case, They appeal to +-¬-E and to the set of correctness-valuations C for signed formulae with the following correctness clauses:[6]

$$(C1) v_c(+A) = T \quad \Leftrightarrow v(A) = T$$
$$(C2) v_c(-A) = T \quad \Leftrightarrow v(A) = F$$

They also define validity for signed formulae:

(VAL) Γ ⊨ α is valid in the case, for every correctness-valuation $\upsilon_c \in C$, if $\upsilon_c(\beta) = T$ for every $\beta \in \Gamma$, then $\upsilon_c(\alpha) = T$.

It appears that C2, VAL and +-¬-E block together Carnap's problem because it says that A and ¬A are both correct. Hence assertion of ¬A, i.e. + (¬A) is correct but since A is correct too, – A cannot be correct. On the basis of C2 and VAL, +-¬-E fails.

On the first appearance, Carnap's problem seems to be solved but actually it just shifts level. For the non-normal valuation can appear at the level of signed formulae (containing + and –): let there be valuation $\upsilon^*_c(\alpha) = T$ for every $\alpha \in WFF_{sign}$. This is the troubleshooting valuation which creates Carnap's problem in the first place now applied to bilateral signed formulae. Both A and ¬A are true and more disturbingly +-¬-E is valid according to VAL: 'the assertability of the premises guarantees the assertability of the conclusion' since for every signed formulae, $\upsilon^*_c(a) = T$. So the bilateral rules for negation do not block Carnap's problem any more. The troublemaking valuation still violates C2 but the appeal to C2 is problematic. Murzi and Hjortland argue that syntactically C2 and NEG are exactly alike and nothing in the bilateral system prevents to view rejection as a special kind of (non-iterative) negation. Hence, C2 is comparable to NEG. So why is it all right to appeal to C2 but not to NEG? (Murzi and Hjortland 2009, 485–486.) I agree that if C2 was the only resource, bilateralism would indeed be in trouble.

6 In my formulation, 'T' stands for epistemically constrained truth which applies only to decidable statements. In broad terms then it coins with correct assertability. It is clear that as an intuitionist, Tennant subscribes to epistemically constrained notion of truth and as I explained above, I assume that Rumfitt subscribes to this too. (See Tennant 1997, 173–177 and Rumfitt 2000, 817–820.)

Nevertheless, Murzi and Hjortland pay too little attention to the bilateral rules for negation. Rumfitt holds that bilateral logic contains a co-ordination principle:

$$\text{(COP)} \quad \frac{+\mathrm{A} \quad -\mathrm{A}}{\bot}$$

Murzi and Hjortland approve this as a bilateral Law of Non-Contradiction

$$\left(\mathrm{LNC}^{*}\right) \alpha,\ \alpha^{*} \vdash \bot,$$

where α^* is the result of reversing the sign of α. (Rumfitt 2000, 816 and Murzi and Hjortland 2009, 485.) The bilateral solution rests on a seemingly trivial observation that because of – -¬-I and – -¬-E, + A and – (¬A) are interdeducible and more importantly because of +-¬-I and +-¬-E, – A and + (¬A) are interdeducible, we have – A ⊣⊢ +(¬A). Hence:

$$+A,\ -A\ \vdash \bot \iff +A,\ +(\neg A) \vdash \bot$$

Thus, the bilateral inferential rules do rule out Carnap's problem for unsigned and signed formulae since the right-hand side of the equivalence says that A and ¬A cannot be both asserted at the same time and the left-hand side rules out valuation $v^*_c(\alpha) = \mathrm{T}$.[7]

In my view, the previous solution works only if we take Tennant's paraconsistent view on ⊥. It is not clear whether Rumfitt actually takes that view but, in the face of Carnap's problem, it might be beneficial. It seems to me that Rumfitt does take some initial steps towards paraconsistency. He says: '[I]t would be perverse to try to assign a propositional content to the expression 'contradiction'. Rather, as Tennant puts it, the expression plays the role of a punctuation mark in deduction' (Rumfitt 2000, 793–794). I take this as a sign that Rumfitt thinks that ⊥ has no propositional content to be clarified with introduction and elimination rules. As a result, Rumfitt admits that intuitionism has the advantage at least in one respect. The intuitionistic rules for negation express in a very direct manner the principle of consistency whereas the bilateral rules do not. For the intuitionistic elimination rule for negation (¬-E above) is a unilateral equivalent of the co-ordination principle (as the above equivalence shows). Therefore, the classical bilateral rules must be coordinated so that they preserve the principle of consistency and they must preserve it in such a way that 'the co-ordination principle (and hence the principle of consistency) holds for complex formulae [+(¬A) and –(¬A)] as well as for atomic ones [+A and –A]' and 'such co-ordination will be necessary if +A and +(¬A) are themselves to be contradictory' (Rumfitt 2000, 815–816). With the troublesome valuation, either side of the equivalence becomes a logical dead-end and that is why the bilateralist can repeat the intuitionistic response.

7 It should be noted that this solution does not depend on the way Hjortland and Murzi formulate the problem. Especially, the current solution does not depend on VAL. In fact, it is likely that VAL needs to be adjusted to accommodate the non-classical consequence in paraconsistent logic. See below.

Conclusion

I agree with Murzi and Hjortland's overall contention. Inferentialism can overcome Carnap's problem. However, there are four qualifications to be made: (i) The analysis of Murzi and Hjortland shows some weak points regarding the connection between absurdity and negation. (ii) Dummett's proposal for absurdity does not clarify the notion falsehood properly. (iii) The paraconsistent solution does better by appealing explicitly to the principle of consistency. Absurdity is a primitive expression of the principle of consistency. Hence, the rules for negation reflect that A and ¬A cannot be asserted at the same time. (iv) A classical inferentialist can solve the problem by adhering to paraconsistency.

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