ON TAKING PREFERENCES SERIOUSLY

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INTRODUCTION

The concept of parliamentary voting procedure is used here to denote the social decision method whereby the socially best alternatives are determined on the basis of the pairwise comparisons of alternatives so that the winner of each comparison is confronted with the next one in the sequence of alternatives and the socially best alternative is the winner of the final comparison. This method is widely used in contemporary parliaments. In this article we shall first take a look at some theoretical results that seem to cast doubt upon the validity of this method as a reasonable way of aggregating individual preferences. Thereafter we shall investigate some alternative voting procedures with a view to determining to what extent, if any, they are exempt from the difficulties exhibited by the parliamentary voting procedure. At first we shall restrict ourselves to binary procedures, i.e. those based on the pairwise comparisons of alternatives. Later on also other types of procedures will be considered.

A major restriction is the assumption of fixed preferences of voters over the alternatives. This assumption is almost universally adopted in social choice literature although its plausibility is questionable particularly in cases where a relatively small number of voters make collective decisions frequently and thus learn to adapt their preferences to those of the others. We are, however, in this article dealing with *institutions* of making social choices. The focus on institutions instead of specific voting situations lends the assumption of fixed preferences more plausibility than is the case when given voting bodies and ballot sequences are considered. In designing viable institutions which purport to be general — i.e. applicable in indefinite circumstances and voting bodies — one cannot rely on specific learning mechanism assumptions although the existence of such mechanisms cannot be denied. Taking particular mechanisms into account would anyhow make it exceedingly difficult to say anything general about voting procedures. Hence we shall make the fixed preference assumption commonly adopted in social choice theory.

THE PROBLEM OF AN EMPTY CORE

The phenomenon called the *Condorcet* paradox has been known for a long time (e.g. Riker & Ordershook 1973, 84). We shall not dwell on that phenomenon here except to point out that in essence the paradox consists of the fact that from a set of individual weak preference orderings (i.e. symmetric, transitive and complete) one may in some circumstances end up with intransitive social preference relations when the parliamentary voting procedure with a simple majority rule is being used. Since such majority cycles are generally deemed undesirable, efforts have been made to place restrictions on the configuration of individual preference orderings so as to exclude the possibility of majority cycles. The best-known of them is the requirement of single-peakedness proposed by *Black* (1958). We know now, however, that in addition to the fact that what is involved in »forcing» any given individual preference configuration to be single-peaked is the imposition upon the voting body of a consent concerning the valuation of some of the alternatives, the single-peakedness condition is extremely unlikely to hold in policy-spaces of more than one dimension if individual preferences are convex (Kramer 1973). On the other hand, the single-peakedness condition is not necessary for the acyclicity of the social preference relation even though it is a sufficient condition for it (see Schofield 1978a).

While the failure of transitivity of the social preference relation may be undesirable, it may still happen that what one is interested in is not the social preference ordering of all alternatives, but just a choice of a subset of »best» ones. When the simple majority rule is applied, the core of a voting game is defined as follows: an alternative x ($\in$ X, the set of alternatives) belongs to the core if and only if there is no y ($\in$ X) such that y defeats x by a simple majority. Now when the parliamentary voting procedure is resorted to, the core alternatives obviously win if everyone votes according to his/her true preferences. We observe that the Condorcet winner – i.e. the alternative that defeats all the others by a simple majority in pairwise comparisons – always belongs to the core.

The problem with the core as a solution concept is that it is often empty. Indeed, the results of *Schofield* show that in more than three-dimensional policy-spaces, the core is generically empty (Schofield 1978b). One is, therefore, led to ask what happens when the core is empty. Obviously there is then no natural equilibrium outcome, i.e. such an outcome that when it is proposed, it can be defeated by no other proposal. But if there is some subset of »reasonable» outcomes in which the social outcome can always be expected to be found (provided the voters are rational), then the absence of the core may not be too much of a problem. The results discussed in the next section, however, render this hope completely unfounded.

McKELVEY'S THEOREM

Suppose that there are n voters in a policy-space and that each voter has an optimum point in the space. Suppose, furthermore, that each voter's preference is Euclidean in the sense that the utility of any point x in the space is a monotonically decreasing function of the distance between x and the voter's optimum x^i. The theorem proven by *McKelvey* (1976) states that under these assumptions and in the absence of a core, it is possible to build an agenda such that starting from an *arbitrary* point x_0 in the space one can make another *arbitrary* point y_0 the winner by the parliamentary voting procedure. In other words, under the above assumptions the agenda-builder can completely control the outcome of the parliamentary voting procedure regardless of the configuration of the individual optima in the space as long as the condition of an empty core is fulfilled (as is generically the case). And yet the winner of each comparison is determined by a simple majority. Hence, the parliamentary voting procedure cannot under previous assumptions guarantee a non-arbitrary outcome when the simple majority rule is applied at each stage. Of course, when a Condorcet winner or, more generally, a core exists, we can predict that it is also the social outcome when the procedure is adopted. As was pointed out, however, it is generically the case that the core is empty in the higher dimensions.

How restrictive, then, are the asumptions of McKelvey's theorem? Prima facie, the spatial model itself may seem unrealistic in as far as it may be difficult to see how a typical political decision making situation could be conceived of as taking place in multi-dimensional real space. Indeed, it may be extremely difficult to locate the decision makers, parties, individuals or whatever, in the space in an unambiguous fashion. Moreover, different analysts might locate the voters differently even when there is agreement concerning the meaning of the dimensions. These observations apparently cast doubt upon the real world relevance of the theorem. Upon closer inspection, however, they are beside the point because the theorem is a possibility result and not a prediction one. Furthermore, for its »application» one does not need to know the exact locations of the voters in the space. Indeed, one does not even need to know what the dimensions stand for. All one needs are the assumptions

1) that there are several dimensions along which the optima of the voters differ,
2) that the ensuing configuration of optima does not have a core, and
3) that the voters in this space calculate in Euclidean fashion.

Assumption 1) certainly is not very restrictive. Assumption 2) describes a typical situation in higher dimensional spaces. Assumption 3), however, needs closer scrutiny.

RELAXING THE RESTRICTIONS ON PREFERENCES

In order to prove theorems about collective decisions in spatial models, some assumptions are needed concerning the voter preferences in the policy space. One assumption which is usually made is that the loss functions of the voters monotonically increase with the distance from the optima. The distance, in turn, can be measured by means of several norms that have this property. In McKelvey's theorem the Euclidean norm is used. This is a severe restriction upon the applicability of the theorem because it can be shown that this assumption implies a certain degree of consensus among the voters. Specifically, if the voter preferences can be represented in the Euclidean fashion in a Re^m-space, it follows that along each of the dimensions the preferences are single-peaked. This, in turn, implies that the voters are unanimous that certain alternatives are not the worst ones, viz. those near the median of the dimension. Thus, the assumption restricts the domain of the theorem to fairly consensual voting bodies only. Of course, one should bear in mind the nature of the theorem: it says that the majority rule is very irregular when the core is empty. If one of the assumptions of the theorem presupposes a given degree of consensus, then one might expect that the arbitrariness of the majority rule holds, a fortiori, in voting bodies with a lesser degree of consensus. Without further ado this conjecture is, however, little more than a guess.

In a recent paper McKelvey (1979) has proven that the arbitrariness of the majority rule does not vanish when the Euclidean loss function assumption is dropped. Indeed, even before that paper *Cohen* (1977) was able to show that McKelvey's theorem is also valid when the indifference curves of the voters are elliptic instead of circular. In terms of the degree of consensus required, Cohen's assumption is no less stringent than McKelvey's original one because no matter which specific convex form the indifference curves have, the preference configuration remains dimension-wise single-peaked as long as the preferences are convex.

In McKelvey's new theorem no convexity assumption is made. The only restrictions on preferences are

(i) that the individual preferences have continuous utility representation, and

(ii) that the voters have no areas of indifference, i.e. the utility functions are not »flat» anywhere in the policy space.

(i) means that for each voter i the following must hold: $\forall x, y \in Re^m : x \, R_i \, y$ if and only if $u_i \, (x) \geqslant u_i \, (y)$, where u_i denotes i's utility function and R_i is i's weak preference relation. The new theorem differs from the earlier one in its method and its strategy of proof: McKelvey now proves that under the above conditions (and two other very mild ones) the frontier F of the set of points reachable from any given x via the simple majority rule is

(a) either empty, or

(b) very restrictive symmetry conditions must hold with respect to individual preferences at F.

The possibility (b) roughly means that almost always the possibility (a) materializes. (a) in turn, is the case whenever either there is a nonempty core or the points reachable from an arbitrary x fill the entire policy space. The latter alternative, of course, means that any point y can be reached from an arbitrary x via the majority rule.

One may note, however, that McKelvey's new result applies to infinite alternative spaces only (e.g. Re^m-space). This, in conjunction with the requirement that there be no flat areas in the individuals' utility functions, may seem to restrict its real world validity. Indeed, some experimental results could be made intelligible if one assumed that individuals have indifference areas instead of curves (see Fiorina & Plott 1978 and Nurmi 1981a). But this observation is not really pertinent if one looks at McKelvey's results with a view to evaluating a social institution, viz. the simple majority rule. Surely, we would wish our institutions to be well-behaved when perfectly rational individuals are involved, even though perfect rationality rarely characterizes the real world decision makers.

In conclusion, then, things look pretty bad for the parliamentary voting procedure with a simple majority rule. In particular, there seems to be no way in which the procedure itself could guarantee that the individual preferences have *any* significance in the determination of social outcomes. In short, the method is utterly insensitive to individual preferences.

SUPPORTING SIZE DECISION SCHEMES

In this section I shall discuss the concept of the supporting size decision scheme (SSDS) following *Barbera* (1979) and, thereafter, relate this concept to a particular binary voting procedure.

Given a fixed set of alternatives X and the set P of strict preference relations of the n individuals, a scheme is defined to be a function from the set of preference relation n-tuples to the measures m over X such that $\sum_{x \in X} m(x) = k$ (constant). When $k = 1$, the scheme is called a decision scheme. Intuitively, a decision scheme is a rule which indicates the probability of each alternative being chosen as the socially best alternative given a certain configuration of individual preferences. If and only if such a scheme has the following additional properties is it called a SSDS:

(1) there exists a n+1 vector of real numbers $A = (a_n, a_{n-1}, \ldots, a_0)$ such that $a_n \geqslant a_{n-1} \geqslant \ldots \geqslant a_0$,

(2) there exists a constant c such that $\forall\, j \leqslant \frac{n}{2} : a_j + a_{n-j} = c$, and

(3) the score s_i of each $x_i \in X$ is obtained as follows:

$$s_i = \sum_{x_t \in X - \{x_i\}} a_{g(x_i, x_t, P)}$$

where $g(x_i, x_t, P)$ denotes the number of those individuals who prefer x_i to x_t when the preference configuration is P. Intuitively, the score is obtained by confronting x_i with each of the other alternatives in X and counting for each pairwise comparison the number r of those individuals preferring x_i to the other alternative. This number r is then used to identify the corresponding a_r in A. The sum of the a_r's thus obtained is the score of x_i.

Now, from the set of m alternatives one can choose $m(m-1)/2$ different pairs. It follows then in virtue of condition (2) above that for $n/2 \leqslant j \leqslant n$: $a_j + a_{n-j} = 2/m(m-1)$ if the scheme is SSDS.

SSDS has some very nice properties (see Barbera 1979). In particular, it can be shown to be anonymous, strategy-proof and alternative-independent. Indeed, any SSDS has these properties and conversely any scheme that has these properties in a SSDS. Anonymity and neutrality are generally deemed good properties (see, however, Plott (1976, 559–560) on symmetry, i.e. neutrality). Alternative-independence means, roughly, the following. Consider four strict preference configurations P^1, P^2, P^3 and P^4. Each configuration has the property that alternatives x_1 and x_2 are adjacent in every individual's preference ranking. Suppose that for each individual i

$$x_1 P_i^1 x_2 \text{ iff } x_1 P_i^4 x_2 \text{ and } x_1 P_i^2 x_2 \text{ iff } x_1 P_i^3 x_2.$$

The scheme is alternative-independent exactly when under these circumstances the change in x_1's probability of being chosen that occurs when P^1 is changed to P^2 equals x_1's probability change when P^3 changes into P^4. In other words, in probability changes the rank order of both x_1 and x_2 with respect to other alternatives does not count, if the scheme is alternative-independent. By and large, alternative independence is also a nice property (see, however, Riker & Ordeshook (1973, 109–114, for discussion) although it explicitly rules out the effects of the preference intensity differences on social choice.

Strategy-proofness is defined by means of manipulability as follows: let $R = (R_1, \ldots, R_i, \ldots, R_n)$ be a n-typle of weak preference relations. Let F be a resolute social choice function $F{:}R^n \rightarrow A$ where A is the set of one element subsets of X and R^n of course the set of all preference n-tuples. R is manipulable by the voter i at R if i strictly prefers F(R′) to F(R) where $R' = (R_1, \ldots, R'_i, \ldots, R_n)$, i.e. R′ differs from R only with respect to i's preference relation (see, e.g. Gärdenfors 1977). Now, F is strategy-proof if it is manipulable by no i at any preference configuration. In other words, if social choice function is strategy-proof, then it is rational for each voter to vote sincerely, that is, according to his/her true preference.

It is often argued that the social choice function should be strategy-proof. Even though this is a nice property, it is perhaps not entirely immoral to advocate social choice functions lacking this property as long as their manipulability is known to each voter. Be that as it may, the *Gibbard-Satterthwaite* theorem is somewhat disturbing in stating that every strategy-proof resolute social choice function is dictatorial if the range of F consists of at least three different alternatives (Gibbard 1973; Satterthwaite 1975). In other words, if one wants a resolute social choice function, then one cannot get a non-dictatorial procedure.

But, of course, dropping the resoluteness property is a way out of this difficulty. Thus, Barbera's theorem on the properties of SSDS shows that there is a trade-off between resoluteness and strategy-proofness. How high, then, is the price paid for strategy-proofness, one could ask. The answer is simple: SSDS produces probability distributions over alternatives instead of single alternatives. The problem that remains is to relate the probabilities with preferences so as to be able to say, e.g. that the most probable alternatives are most preferred by the voting body. One could think of circumventing the problem by resorting to random devices. Thereby one could guarantee that the procedure is sensitive to individual preferences in the long run. Perhaps this is the only relevant consideration in the institutional design, but one should pay due attention to the fact that in individual cases, i.e. in specific decision making situations, the procedure can be entirely unresponsive to individual preferences. Consequently, it would be difficult to apply e.g. in electing public officials let alone presidents. Let us therefore consider a procedure that bears some resemblance to SSDS's but does not end up with probability distributions. It will turn out that the price at which the non-random nature is bought is high indeed.

THE MAXIMIN SUPPORT SET

Suppose that the alternative set X is finite and of cardinality m. Then we could form a mxm matrix $[r_{ij}]$ where r_{ij} denotes the degree to which x_i is preferred to x_j, with $r_{ij} = 1$ indicating a definite preference of x_i over x_j, $r_{ij} = ½$ indifference and $r_{ij} = 0$ a definite preference of x_j over x_i (see, Bezdek et al. 1978; Nurmi 1981b). Let the matrix be formed by performing pairwise comparisons between all pairs of alternatives (x_i, x_j) so that

$$r_{ij} = 1/n \sum_{k=1}^{n} b_{ij}^{k}, \text{ for } i \neq j, \; r_{ij} = 1, \text{ for } i = j,$$

where $b_{ij}^k = 1$ if the voter k prefers x_i to x_j and $b_{ij}^k = 0$, otherwise.

Now, *Skala* (1978) has shown that the $[r_{ij}]$ matrix interpreted as the social preference relation is

(1) pseudo-transitive in the sense that $r_{ij} + r_{jh} - 1 \leqslant r_{ih}$,
(2) pseudo-asymmetric in the sense that $r_{ij} + r_{ji} \leqslant 1$, and
(3) irreflexive.

Skala, moreover, shows that the conditions of the *Arrow* Impossibility Theorem are consistent if we are satisfied with the social preference relation that has the properties (1)–(3).

But very few are satisfied with the above type of social preference relation because it gives no clue in general as to how to make social choices. I have elsewhere proposed a procedure that uses the above $[r_{ij}]$ matrix as input and identifies a non-empty subset of X as the set of socially best alternatives (Nurmi 1981b). In the present context I shall somewhat modify this procedure. Before going into that, let us observe that the $[r_{ij}]$ matrix can always be transformed into a SSDS by simply defining the entries of the matrix

$$r_{ij} = \sum_{k=1}^{n} b_{ij}^{k}.$$

The scores of the alternatives could, then, be computed as

$$s_i = \sum_{j=1}^{m} r_{ij} / \sum_{i=1}^{m} \sum_{j=1}^{m} r_{ij}.$$

This way of defining the scores guarantees that the procedure is a SSDS (cf. the definition above). Hence, if we are satisfied with a procedure resulting in a probability distribution over alternatives, we could resort to this method of defining the scores. However, if random processes cannot be used, there is no guidance as to which alternatives are socially best. In order to overcome this problem we define for each $x_i \in X$:

$$v(x_i) = \min_j r_{ij},\ i \neq j.$$

Now, we say that x_s belongs to set X_M of maximin support iff

$$v(x_s) = \max_i \min_j r_{ij}.$$ (cf. Kramer 1977).

The procedure whereby X_M is used to determine the socially best outcomes has several properties to recommend it.

Proposition 1. The maximin method is decisive in the sense that for all n-tuples of individual preferences, it yields a nonempty set of social outcomes.

This can be seen by noticing that on each row of the $[r_{ij}]$ matrix there is a minimum entry.

Proposition 2. The method chooses a Condorcet winner if one exists.

Proposition 3. The method chooses a core outcome if one exists.

Proof. Since Proposition 3 implies Proposition 2, we shall consider Proposition 3 only. Suppose there is a core alternative x_k. It follows then that (1) $\nexists\, x \in X$: $x\ r(M)\ x_k$ where r(M) means that the element on the left side defeats by a simple majority the element of the right side. On the other hand, (2) $\forall x_i \neq x_k\ \exists\, x \in X$ such that $x\ r(M)\ x_i$. Now (1) is equivalent to the following: $\min_j r_{kj} \geqslant n/2$. (2), on the other hand is equivalent to the following: $\min_j r_{ij} < n/2$. That is, $x_k \in X_M$ and $x_i \notin X_M$. Q.E.D.

Along with these obviously nice properties the maximin method has many undesirable ones. The first is inconsistency. The consistency of a procedure is defined as follows (see Young 1974; Mueller 1979, 62). Let there be two voting bodies N_1 and N_2 which make an independent social choice according to method K from the set X. Let the choice of the first body be X_1 and the choice of the second one X_2. Suppose that $X_1 \cap X_2 \neq \emptyset$. If the choice of $N_1 \cup N_2$ also using K is identical with $X_1 \cap X_2$, the procedure is said to be consistent.

Although the consistency property in this sense would undoubtedly be a nice theoretical property for a collective decision-making procedure, it is not essential for a social institution with a fixed number of members. After all, it is known that in real-world decision making bodies, the entry of new members can sometimes increase the voting power of the old members as measured by power indices (see, Brams & Affuso 1976). Inconsistency is an analogous »paradox» of the maximin rule. Not all procedures are, of course, inconsistent. For instance, the *Borda* count is a consistent method.

Proposition 4. The maximin method is inconsistent.

Proof. By way of a counterexample. Let the preference profile of group N_1 over the set $X = \{x_1, x_2, x_3, x_4\}$ be the following:

person 1	person 2	person 3
x_1	x_4	x_2
x_3	x_1	x_3
x_2	x_3	x_4
x_4	x_2	x_1

and for group N_2 the following (the numbers refer to persons):

1 and 2	3	4	5	6	7
x_1	x_3	x_2	x_4	x_4	x_4
x_2	x_4	x_3	x_3	x_3	x_2
x_3	x_1	x_1	x_2	x_1	x_1
x_4	x_2	x_4	x_1	x_2	x_3

The matrix of pairwise comparisons is then the following for N_1:

	x_1	x_2	x_3	x_4	min
x_1	–	2	2	1	1
x_2	1	–	1	2	1
x_3	1	2	–	2	1
x_4	2	1	1	–	1

and for N_2:

	x_1	x_2	x_3	x_4	min
x_1	–	4	3	3	3
x_2	3	–	4	3	3
x_3	4	3	–	4	3
x_4	4	4	3	–	3

N_1's choice set X_1 is then $\{x_1, x_2, x_3, x_4\}$ and N_2's choice set $X_2 = \{x_1, x_2, x_3, x_4\}$ as well. However, when the groups are combined, the pairwise comparison matrix is the following:

	x_1	x_2	x_3	x_4	min
x_1	–	6	5	4	4
x_2	4	–	5	5	4
x_3	5	5	–	6	5
x_4	6	5	4	–	4

The choice set of N is then x_3, thus showing that the maximin method is inconsistent. Q.E.D.

The social choice method has the cancellation property if whenever for any n-tuple of preference relations over X and for every pair $(x_i, x_j) \in X \times X$, the number of voters preferring x_i to x_j equals the number of voters preferring x_j to x_i, then the social choice is X. The following proposition is obvious.

Proposition 5. The maximin method has the cancellation property, if no abstentions are allowed.

Another consistency-type property that characterizes some procedures is called the weak axiom of revealed preference (WARP). Suppose that $X' \subset X$ and that the preference configuration of the n voters is the same when the choice is made from X′ as when it is made from X. Denote by A (B, respectively) the choice set when X (X′) is considered. Now, if $X' \cap A \neq \emptyset$ implies $\{x \mid x \in X' \text{ and } x \in A\} = B$, then the procedure satisfies WARP.

We noticed that the maximin method does not satisfy the first type of consistency. That it does not satisfy WARP either can be seen from the following.

Proposition 6. The maximin method does not satisfy WARP.

Proof. Again by way of a counterexample. Consider the following preference profile over $X = \{x, y, z\}$:

1	2	3
x	z	y
y	x	z
z	y	x

The choice set is obviously $\{x, y, z\}$ as every alternative gets the minimum of 1 vote in pairwise comparisons. Consider now the subset $X' = \{x, y\}$ of X. The maximin winner of this subset is obviously x, while the intersection of X′ and the choice set of X is $\{x, y\}$. Hence WARP is violated. Q.E.D.

Clearly the minimum number of votes a given alternative gets when it is confronted with all other alternatives depends on the entire set of alternatives. Therefore, if some alternatives are infeasible, their removal from consideration can change the choice set under the maximin method. Thus, whether or not an alternative is chosen depends partly on what other alternatives are available. The following expresses this property of the maximin method (see Plott 1976, 518).

Proposition 7. The maximin method violates the value/feasibility separation.

One further consistency-related property should be mentioned, viz. path-independence. Let R be a fixed preference profile. We denote by C(X, R) the social choice set resulting from the application of a fixed voting procedure to the set X of alternatives, when R is the preference profile. If now for all partitionings X_1, X_2 of X: $C(X, R) = C(C(X_1, R) \cup X_2, R)$, then the procedure which realizes C is path-independent. In other words, if the social choice set remains invariant under various partitionings of the alternative set, then the procedure is path-independent.

Proposition 8. The maximin method is not path-independent. This proposition has been discussed elsewhere (Nurmi 1981c).

We conclude this section with the following propositions.

Proposition 9. The maximin method is manipulable.

Proof. Consider the following preference profile over $X = \{x, y, z, v\}$

1	2	3	4
x	y	x	v
y	z	y	z
z	x	z	x
v	v	v	y

The pairwise comparison matrix is then the following:

	x	y	z	v	min
x	–	3	2	3	2
y	1	–	3	3	1
z	2	1	–	3	1
v	1	1	1	–	1

The maximin choice consists of x only. By misrepresenting his/her preferences as zyxv instead of yzxv, person 2 can bring about the following matrix:

	x	y	z	v	min
x	–	3	2	3	2
y	1	–	2	3	1
z	2	2	–	3	2
v	1	1	1	–	1

Now the choice is $\{x, z\}$, clearly a preferable outcome for person 2. (We shall return to this assumption of preference shortly). Hence, by misrepresenting his/her preferences a voter can bring about a better outcome than by voting sincerely. Q.E.D.

Now manipulability is definitely not a desirable property of a voting procedure, but it seems that it characterizes all voting systems currently in use. A more fatal flaw of the maximin method is the following.

Proposition 10. The maximin method can choose a Condorcet-losing alternative.

In other words, in some cases where an alternative can be found that would be defeated in pairwise comparisons by all the other alternatives, the maximin method would yet choose this alternative. This can be seen from the following example.

person 1	person 2	person 3
x	z	y
v	v	z
y	y	v
z	x	x

The pairwise comparisons:

	x	y	z	v	min
x	–	1	1	1	1
y	2	–	2	1	1
z	2	1	–	2	1
v	2	2	1	–	1

Thus, x which is the Condorcet-loser, is chosen along with the other alternatives.

OTHER BINARY PROCEDURES

It seems that when trying to avoid the systematic use of random devices in making social choices we have to pay a high price at least as far as the maximin method is concerned: it does away with lotteries but ends up with other difficulties as shown in the above propositions.

By and large the binary methods – i.e. methods based on the pairwise comparisons of alternatives – do rather badly with respect to choice set invariance criteria (WARP, consistency, path-independence). Their strong points are the Condorcet-criteria. Thus, for example, the parliamentary voting procedure chooses a Condorcet winner when one exists and never chooses a Condorcet loser. In terms of these two Condorcet criteria the parliamentary voting procedure dominates the maximin method. However, the parliamentary voting procedure is not *Pareto*-optimal whereas the maximin method is, as has been argued in *Nurmi* (1981c). These remarks are based on research reported at greater length elsewhere (Nurmi 1983; see also Richelson 1979 and Straffin 1981). Hence it seems that there is not much point in switching from the parliamentary voting procedure to the maximin method: one would gain Pareto-optimality but take the risk of sometimes choosing a Condorcet-loser. Quite a few people would agree that the.switch is not worth making.

Would it then be possible to find a method that is both Pareto-optimal and never chooses a Condorcet-loser and, moreover, has the good properties that the maximin method and the parliamentary voting procedure have in common, viz. the choice of the Condorcet-winner whenever one exists and monotonicity (i.e. the property that if an alternative wins when a given procedure is used, and then some individuals change their minds so as to lift the winner higher in their preference orders, then the same alternative would still win if no other changes are made in the preference orders)? Yes, it would. It is a procedure designed by *Copeland*. The method is based on a scoring function. The Copeland-score of an alternative x_j is determined by taking the number of alternatives that x_j defeats and subtracting from this number the number of those alternatives that defeat x_j. The alternative with the largest score is the Copeland-winner.

ONE-STAGE PROCEDURES

But with respect to the choice set invariance properties – WARP, consistency and path-independence – the performance of Copeland's method is equally unimpressive as that of the maximin method and the parliamentary voting procedure. None of these satisfies any of the criteria mentioned. As I have shown elsewhere the best-known multi-stage procedures – the plurality runoff, *Nanson's* Borda elimination, the preferential voting methods of *Hare* and *Coombs* as well as Black's method – are not superior to Copeland's procedure. As a matter of fact, Copeland's method dominates all of them except Black's method when the Condorcet-criteria, monotonicity, Pareto-optimality, WARP, path-independence and consistency are considered (Nurmi 1983; see also Richelson 1979 and Straffin 1981). Therefore, if one is interested in the rationality criteria – i.e. monotonicity and Pareto-optimality – and choice set invariance criteria, one should look at one-stage procedures.

One-stage procedures are based on a simultaneous consideration of all the alternatives. Perhaps the most common of these procedures is the plurality principle. The Borda count and the approval voting are also pretty well-known. The former was introduced roughly two centuries ago by *Jean-Charles de Borda* (see deGrazia 1953) and the latter recently by *Brams* and *Fishburn* (1978; 1981). It turns out that neither the plurality procedure nor the Borda count satisfy WARP. On the other hand, the Borda count never chooses a Condorcet-loser, whereas the plurality method does not necessarily exclude such a choice. The best buy in terms of the choice set invariance criteria would, however, be approval voting: it dominates – with respect to these criteria – both the plurality procedure and the Borda count. Moreover, it dominates all the procedures mentioned above as far as these criteria are concerned. This is because approval voting is path-independent and consistent and satisfies WARP. Moreover, it is monotonic and in a sense Pareto-optimal (Nurmi 1983). However, it fails on both Condorcet criteria; in other words, it does not necessarily choose the Condorcet-winner when one exists and can choose the Condorcet-loser when one exists.

THE PROBLEM OF STRATEGY

Manipulability

As was pointed out above, the theorem independently proven by Gibbard (1973) and Satterthwaite (1975) states that when the number of alternatives is at least three all non-trivial resolute social choice functions are either manipulable or dictatorial. The resolute social choice functions are characterized by the property that their range is a set consisting of single alternatives only, i.e. no ties can result from resolute social choice procedures. Now if we consider those voting procedures that have done reasonably well in the light of Condorcet's, rationality and choice set invariance criteria – i.e. Copeland's and Black's methods along with the Borda count and approval voting – we notice that none of them is resolute. In other words, ties of two or more alternatives can result from each of them. Hence, the Gibbard-Satterthwaite theorem is not directly applicable. Let us now focus on the manipulability of the most promising voting procedures or – to be more exact – on the manipulability of the choice functions which these procedures realize.

Copeland's procedure is manipulable as can be seen from the following sincere preference profile:

1 person	2 persons	1 person	1 person
x	z	y	y
w	x	w	z
y	w	x	w
z	y	z	x

The Copeland scores are x:1, y:−1, z:1 and w:−1. Hence, the result is a tie between x and z. Now if the person whose least preferred alternative is x insincerely indicates that his/her preference ordering is zywx, the Copeland scores change: x gets 1, y −3, z 3 and w −1. The winner is now z. Hence by misrevelation of his/her true preference the person is able to bring about an outcome that he/she prefers different to the one resulting from the revelation of his/her true preferences.

Now when saying that z is preferred to a tie between x and z by the person in question, we are making an assumption that is a milder version of the monotonicity in prizes of *Harsanyi* (1977, 33). The latter states that if A is preferred to B, then the lottery (A, p; C, 1−p) is preferred to the lottery (B, p; C, 1−p) where $0 < p < 1$. ((A, p; C, 1−p) denotes the lottery in which the probability of prize A is p and the probability of prize C is 1−p). As a matter of fact we are committing ourselves to a milder assumption, viz. that A is preferred to the lottery (A, p; B, 1−p) when A is preferred to B. To apply this assumption to our case, we just let A = z and B = x.

As for the manipulability of the Borda count, there is some anecdotal evidence that J.-C. de Borda himself was aware of this weakness of his method (Mascart 1919, 130). An example will show that the Borda count is indeed manipulable. Consider the following profile of true preferences:

1	2	3
x	y	z
y	z	x
v	v	y
z	x	v

Here the numbers identify persons. The Borda scores using de Borda's original scoring, i.e. 4 points for the first rank, 3 for the second etc., are as follows: x 8, y 9, z 8 and v 5. The winner is thus y. Person 3 can misrepresent his/her preferences as xvzy and change the scores as follows: x gets now 9, y 8, z 6 and v 7 points. Now x wins. Person 3 prefers x to y and thus the procedure is not strategy-proof.

Turning now to Black's method, we notice first that in the previous example there is no Condorcet winner. Therefore, Black's method chooses the Borda winner y. Now by misrepresenting his/her preferences again as xvzy, person 3 renders x the Condorcet winner. Black's method then chooses x which is preferred to y by person 3 as we noticed. This shows that Black's method is also manipulable.

So is approval voting. This is so because the plurality method, a special case of approval voting, is manipulable. Consider the following example.

2 persons	3 persons	2 persons
x	y	z
y	z	x
z	x	y

If everyone votes according to his/her true preferences and the plurality method is used, the winner is obviously y. This alternative is the worst one for the two persons on the right. By voting as if his/her true preference were xzy, one of these two persons can create a tie between x and y. By the principle of monotonicity in prizes, a tie is preferred to y by the person in question. Hence, the plurality method is manipulable. The same example shows also that the approval voting is manipulable if each voter approves one alternative only.

Truncation of Preferences

None of the most promising voting procedures thus turns out to be strategy-proof. Indeed, all of them are manipulable by some *individual* in some situation. A fortiori they are also manipulable by coalitions of voters. Let us now focus on another type of manipulability which has received relatively slight attention, viz. the truncation of preferences (Brams 1982a; Fishburn & Brams 1983). Brams shows that one fairly common type of preferential voting, viz. Hare's method, is vulnerable to the truncation of preferences. In other words, a voter can sometimes benefit from not indicating any preference at all for some alternatives. In all voting systems which utilize the preference orderings of voters as inputs this kind of strategic behaviour is relevant. All the methods discussed in previous section can be implemented by using preferences as input data. Therefore, the truncation of preferences deserves some consideration.

Copeland's procedure is vulnerable to »the truncation paradox», i.e. a voter may benefit from not revealing any preference at all for some alternatives. Since we are dealing with preference order inputs, we assume that the truncation of preferences means that the voter gives the order of his/her most preferred alternatives only and indicates no preference with respect to others. This means that if x and y both belong to the »indifference set» of voter i, the preferences of other not indifferent voters determines whether x beats y. On the other hand, if only x belongs to i's indifference set, then obviously i's preference of y over x is taken into account. Let us now take a look at the following example.

1 person	2 persons	1 person	2 persons
x	z	y	y
w	x	w	z
y	w	x	w
z	y	z	x

The Copeland scores are then: x −1, y 1, z 1 and w −1. Hence there is a tie between y and z. Obviously the result does not please the left-most person very much. He/she can do better, however, by truncating his/her preferences so that only x is indicated as the most preferred alternative. The new scores are then: x−1, y 2, z 1 and w −2. Now y, which is preferred to the z-y tie by the person in question according to the monotonicity in prizes principle, wins. Incidentally, if *two* alternatives would be chosen by Copeland's method the benefit from truncation would be even more marked: in the sincere case y and z, i.e. the least-preferred alternatives of the left-most person, would be chosen. On the other hand, if the person gives the above truncated preference only, x and y would be chosen. Notice that x is the most-preferred alternative of the person in question.

Now, the Borda count is obviously vulnerable to the truncation of preferences if the voter is allowed to give his/her Borda scores to the most-preferred alternatives only and to give the score zero to the others, provided that the scores range from 0 to n−1 where n is the number of alternatives. The following case shows that the Borda count has this drawback.

1	2	3
x	z	y
y	y	z
z	w	w
w	x	x

Assuming that the available scores are 0, 1, 2 and 3, the result is: x 3, y 7, z 6 and w 2. If now person 2 decides to truncate his/her preferences so as to give z 3 points and zero points to the other alternatives, the new Borda scores become: x 3, y 5, z 6 and w 1. Now z, person 2's most-preferred alternative, wins. One could object that this notion of truncation is different from the previous one in doing more than just assigning the same score to all alternatives for which the preferences are truncated, viz. in fact treating all these alternatives as the least preferred ones. Perhaps a more plausible interpretation of preference truncation would be to assign the median Borda score to all alternatives in the indifference set. A glance at the previous example reveals that even so the Borda count is vulnerable to the truncation paradox. If now person 2 again truncates his/her preferences as above, he/she assigns 1 point to y, w and x. The new scores now are: x 4, y 6, z 6 and w 2. Obviously the y-z tie is preferred to y by person 2.

Black's method also has this drawback. This can be seen from the previous example. Since y is the Condorcet winner it is chosen by Black's method. If person 2 truncates his/her preferences in the fashion described above, y is no more the Condorcet winner. Thus the procedure chooses the Borda winner and the result is a y-z tie as was pointed out above.

The truncation of preferences has to be given yet another interpretation when approval voting is considered. It would be natural to say that a voter truncates his/her preferences whenever he/she votes for a most-preferred proper subset of those alternatives that he/she approves. Let us consider again an example.

1	2	3
a	a	b
b	b	c
c	c	a

Voters 1 and 2 have identical preferences over $X = \{a, b, c\}$. Assume now that they both approve a and b. If they vote accordingly and voter 3 approves b only, the result is that b will be chosen. If voters 1 and 2 truncate their preferences so as to vote for a only the behaviour of voter 3 remaining the same as previously, the alternative a wins. Alternatively, one could show that one of the voters 1 and 2 could create an a-b tie by truncating his/her preferences. Both cases show that approval voting is also subject to the truncation paradox.

Strategic Information

All democratic voting systems aim at finding out the preferences of the voters. If a system is manipulable, there is no assurance that true preferences are revealed. A modern and nowadays quite routine feature of political life, opinion polls, presents yet another challenge to the voting systems. As Brams

(1982) points out, the polls create in effect a runoff system even when the voting system formally is a one-stage procedure. A person whose first preference has a poor showing at the polls is likely to misrepresent his/her preferences so as to help the election of the alternative that he/she most prefers among the »feasible» ones. The problem we now address ourselves to is the following: how is the poll information likely to affect the preference revelation in Copeland's, Black's and Borda's methods as well as in approval voting?

Quite obviously the poll information calls for similar strategic modifications of the true preferences in Copeland's, Black's and Borda's systems so that the most-preferred »feasible» alternative is ranked first, while the least-preferred of the feasible ones is ranked last, provided that the poll information indicates a fairly close race between the two alternatives. With this modification the voter can be sure of guaranteeing as many victories and as few losses for his/her most-preferred alternatives as he/she possibly can in pairwise comparison when Copeland's procedure is used. The same modification guarantees the smallest possible number of victories and the largest number of losses to the least-preferred feasible alternative.

When the Borda count is used these modifications lead to the largest (smallest, respectively) Borda score to the most-preferred (least-preferred) feasible alternative that the voter can possibly give.

It has been known for two centuries that the Condorcet winner may not be one of the two alternatives that have the best showing at the polls (if the first preferences only are focused upon). Hence the fact that there is a Condorcet winner does not diminish the possibilities of similar strategic modifications as above. As Black's method chooses the Borda winner when no Condorcet winner exists, we notice that the same kind of strategic modifications can be made when Black's procedure is resorted to.

Approval voting is also sensitive to poll information. In this case, however, the modification called for is of a different nature (Brams 1982): all one needs to do is to contract or expand the set of approved alternatives so that the more preferred one of the poll-favourites is in the approved set, while the other one is not. The difference between this modification and the ones discussed earlier in this section is interesting: while the latter necessarily require insincerity in preference revelation, the former (i.e. the modification in approval voting) does not. That is, a voter can still vote for all the alternatives that he/she prefers to the best feasible alternative. His/her preference ordering may remain the same, while the »boundary» between the approved and disapproved alternatives changes due to the strategic utilization of the poll information.

CONCLUDING DISCUSSION

Could one then on the basis of the preceding analysis make practical recommendations concerning the optimal voting system? The answer depends on the view one has of the relevance of the criteria in the light of which the voting systems have been evaluated. Moreover, one has to bear in mind that not all voting systems have been discussed. But with these reservations in mind it seems pertinent to look at the implementation problematique of the most promising voting systems we have touched upon.

In their recent work Brams and Fishburn (1978; 1983) have made a strong case for approval voting. Indeed, the rationality and choice set invariance criteria speak clearly in favour of this voting system. Also the possibilities for strategic manipulation are fewer in approval voting than in plurality or plurality runoff methods both of which are widely used in group decision making. There is, however, a serious drawback in approval voting, viz. the fact that it does not necessarily choose the Condorcet winner when one exists even when everyone votes according to his/her true preferences. Moreover, the method can choose the Condorcet loser when one exists. In this latter respect approval voting does worse than the plurality runoff which can never choose the Condorcet loser. With respect to the Condorcet winner criterion all three – approval voting, plurality and plurality runoff methods – do equally badly.

When compared with the more promising voting methods – Copeland's, Black's and Borda's – the performance of approval voting is still impressive as far as the rationality and choice set invariance criteria are concerned. We noticed in the preceding that approval voting fares similarly as the above three voting procedures in terms of manipulability, truncation paradox and strategic information induced preference modifications. However, when it comes to the Condorcet criteria – winner or loser – each one of the three systems satisfies at least one of them. Indeed, Copeland's method satisfies both. Approval voting, as we noticed, is consistent with neither of the Condorcet criteria.

The recommendations ensuing from this analysis are twofold

(1) if the emphasis is on the Condorcet criteria and the choice set invariance criteria are given secondary importance only, then Copeland's method would seem optimal, and

(2) if all criteria are deemed equally important, then the approval voting should be chosen.

One might, of course, try to combine the nice properties of various procedures in an effort to design an optimal voting method. It turns out, however, that the »hybrid» procedure may have properties that its constituents do not have and vice versa. Indeed, it can happen that even when the hybrid procedure consists of repeated applications of a given constituent procedure, it has different properties than the constituent method. A case in point is Nanson's Borda elimination procedure which satisfies the Condorcet winning criterion while the Borda count does not satisfy it. The reason is obviously that in Nanson's procedure the Borda scores are used eliminatively while in Borda count they are used to determine the winners.

Similarly when different procedures are combined we might get somewhat counterintuitive results. One might, for example, wish to strengthen one weak point of the approval voting, viz. the Condorcet winning criterion by constructing a method which necessarily chooses the Condorcet winner when one exists. However, if one combines the approval voting with this criterion, the resulting hybrid method does not have WARP any more (even though the approval voting has it). The following Condorcet cycle illustrates this:

person 1	person 2	person 3
x	y	z
y	z	x
z	x	y

Suppose that each person approves his/her first preference only. Hence as there is no Condorcet winner, there is a tie between x, y and z. Consider now the subset $\{x, y\}$ of alternatives. In this subset there is a Condorcet winner, viz. x. As WARP requires that each alternative which is among the winners in a set of alternatives, should be among the winners in every subset to which it belongs, we see that WARP is violated.

The same example can also be used to show that the hybrid procedure fails to satisfy path-independence. If we first determine the winners in the subset consisting of x and y, the winner is x. Considering then the set $\{x, z\}$ we notice that z wins. If the entire set is considered, the winners are x, y and z. Hence, the procedure is not path-independent either.

Also consistency property vanishes in the hybrid method. Consider the following case:

	1	2	3		4 persons	5 persons	2 persons
	x	z	y		x	z	y
Group N:	y	x	z	Group N':	z	x	x
	z	y	x		y	y	z

Assuming again that in each group every person approves only his/her first ranked alternative, there is a tie between x, y and z in N, while x is the Condorcet winner in N'. In N∪N' there is no Condorcet winner. Therefore, the approval voting is applied with the result that z is the winner. Clearly the procedure is inconsistent.

These two properties are sufficient to show that combining two methods with intuitively desirable properties may result in a genuinely new method. Whether the new method possesses the properties of its constituents is a contingent matter to be determined in casu.

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